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OPTIMAL BLACK START RESTORATION IMPLEMENTATION USING DISTRIBUTED ENERGY RESOURCES

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Abstract - **Increasing energy demands in all sectors have driven new and innovative solutions to tackle the evolving energy challenges. Distributed energy resources (DERs) integration and deployment to support the grid during periods of peak demands are on the increase. However, uncontrolled integration of some renewable energy sources into the grid may affect the system inertia, culminating to a frequency shift. In this study, the contribution of DERs in grid energisation and stabilisation during black start restoration (BSR) operations is investigated. The BSR problem is formulated and executed as a dynamic optimisation problem. This enables the effective coordination of the dispatching actions of the DERs along with the switching actions of remote controllable switches (RCSs) over multiple decisions time steps. To satisfy the nature of the developed BSR method, the Mixed-Integer Linear Programming (MILP) technique was adopted and used in the performance analysis of the Nigerian 330 kV 48-bus system modelled for this study. The simulation of the MILP model was achieved in MATLAB® using the IBM CPLEX™ solver. The BSR sequence and the development of a viable restoration strategy were also actualised on the Nigerian 330 kV 48-bus system. The modelled simulated test system and the analysis resulting from the loading of a real network verified the effectiveness of the developed model.**

Keywords – **Black start restoration, distributed energy resources, dynamic optimisation, grid stabilisation, power system, power restoration sequence**

I. Introduction

The era of global over dependence on oil for energy related services is slowly but gradually grinding to a halt. Around the world, with increased evidence of the devastating effects of global warming, various partial or total renewable energy (RE) transitioning timelines are being set, culminating to the increasing level of RE penetration and integration. For instance reports from [1, 2] indicates that many countries were generating more than 20% of their electricity from solar and wind hence, making solar photovoltaic and wind clearly their main stream options in the power sector. The burning effect of climate change resulting from greenhouse gas emission and other such pollutions have continued to impact negatively on the environment. Statistically, research has shown that human activities relating to energy production and consumption are the leading cause of greenhouse gas and other related emissions [3]. With edge cutting developments in the field of RE technologies, the cost of energy from RE sources is increasingly becoming competitive with that of the fossil-based power plants. For instance, [1] reported that renewable power cost, when compared to conventional fossil fuel-fired power plant was increasingly becoming cost competitive. Specifically, in some locations, the energy from wind and solar photovoltaic (PV) plants were more economical than power from fossil fuel-fired plants. By the end of 2024, the total global renewable power capacity was about 4,448 GW with solar PV and wind accounting for 96.6% of net RE additions [1].

Consequently, the use of DERs as an alternative energy source or in conjunction with conventional energy sources is on the increase. For example, [2] noted that during blackout scenarios, some DERs are used in the provision of power to support

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critical facilities. With such capacity and reliability comes the quest to integrate DERs in the provision of black start services during power system restoration (PSR). The contributions of DERs both renewable and non-renewable in the provision of alternative source of power or in grid/off grid applications especially at the LV distribution level are frequently being reported. For instance, studies in [4 - 6] observed that recent technological research has shown the potential of employing DERs along with RCSs in the provision of black start services. Furthermore, other studies also reported the progress made in the deployment of electric buses in the distribution system to facilitate and enhance the system resilience [7]. All of these measures are strong indications pointing to the possibility of DERs deployment in the provision of black start services.

Statistically, renewables-based generation was observed to have increased significantly, with a resultant share of renewables in electricity generation mix projected to increase from 41% in 2024 to 47% in 2027 [2, 8]. This according to [8] was attributed to the double-digit percentage increase from wind power and a major boost in the solar photovoltaic (PV) output from projects carried out in previous years. As the world population continues to increase, the global energy demand is also expected to rise and meeting the growing energy demand in a sustainable manner has remained one of the greatest challenges of the 21st century. Increasing the energy supply to meet the growing demand is one of the four elements of sustainability highlighted by [9 - 11]. As observed by [8], to achieve a sustainable energy system requires the provision of affordable energy to everyone from an energy source that is devoid of high risk. It also involves improving the living standards for millions the world over.

In Nigeria for instance, the application DERs in supporting energy resilience in a localised region was demonstrated in Mokoloki, a community in Ogun state. The Nigeria's first under-grid mini-grid pilot project lunched in February 2020, has the capacity to provide constant electricity to power the health clinic in Mokoloki community thereby keeping medicines stored in the clinic's refrigerators safe [12]. The resilience of the mini-grid project especially during the very difficult times and uncertainties resulting from the global COVID-19 pandemic laid credence to the credibility and viability of DERs in providing smart solutions to local energy system, which can enhance a sustainable future. Besides enhancing the services of the local healthcare delivery, other business opportunities are created which in turn trigger development and sustained economic growth.

II. Literature Review

Different authors [13 - 24] have carried out several studies on black start restoration methods. These studies gives useful insights regarding the various merits and demerits of some of the restoration approaches adopted. To understand why the different restoration methods or a combination of methods: heuristics, meta-heuristics, expert systems, mathematical programming, and multi-agent is required, some of the methods suggested are briefly reviewed.

To serve critical loads in a secondary distribution system network, the authors in [25] proposed a look-ahead service restoration method that was based on DGs. Their assumption was based on the availability of a centrally controlled loads, DGs, and hierarchical control infrastructure. To address the critical load restoration problem, [26] developed a two-stage heuristic method by modelling a critical load problem as a chance-constrained stochastic program. First, a strategy table was created using restoration sequence, before solving the linear integer optimisation problem to obtain the restoration strategy. The ingenuity of the adopted approach seems to have been questioned due to the lack of practical demonstration in real systems to guarantee its effectiveness. A three-stage service restoration approach involving first, establishing a feasible restoration tree from microgrid to critical loads, before incorporating the reserve capacity as constraints of the restoration problem and lastly deploying emergency power supply vehicles was proposed by [27]. By adopting this method, the system resilience was evaluated by the non-parametric kernel density estimation and numerical simulations conducted to verify the effectiveness of this approach. Reference [28] proposed a heuristic

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method aimed at obtaining the allocation plan by solving a mixed-integer linear program. The method was validated in various scenarios using electric buses as a kind of resource by performing numerical simulations performed on the IEE 123-node feeder system.

The four-step approach namely; restoration tree construction, load group formation, solving the critical load restoration problem as a maximum coverage problem, and lastly determining the restoration action was proposed by [29] as a method of grid restoration for critical load on the distribution feeder using a microgrid structure. Using the proposed method, the maximum coverage problem that presents as a linear integer program (LIP) was solved. The restoration method was applied to a practical distribution system and the outcome appeared to have been satisfactory. Similarly, a service restoration problem in a microgrid was solved by [30] by simply selecting the best possible arrangement of power network configuration. Lagrangian technique and dynamic programming was applied to obtain an analytical sub-optimal but efficient solution to the problem. The simulated results did prove useful in providing solutions very close to the optima. The formulation of the service restoration problem as a Mixed Integer Non-Linear Problem (MINLP), using the mathematical programming approach was proposed by [14, 16, 19]. Using this method of service restoration, optimal solutions could be achieved. However due to the expansion problem, the computational time would normally exceed the practically allotted time. To address the shortfall of the mathematical programming approach, [23, 24] proposed using a multi-agent system. Using heuristic method, a restoration scheme designed to minimise feeder losses while facilitating faster service restoration was proposed by [15, 17]. The proposed method, unlike previous ones, relied on branch power flow rather than on current flow. Optimal configuration and reduced computational burden culminating to better restoration plan was reported using this method.

A knowledge-based technique involving the representation of expert knowledge as rules by the application of (IF-THEN statements) and the use of inference engine to infer from such rules was adopted by [13, 20] to activate a restoration plan with appropriate switching sequence. The effectiveness of this approach in tackling in-service restoration problem is widely documented. However, the associated high maintenance cost of large expert system is a major drawback. In a related study, adopting the meta-heuristics approach, [21, 22] used a combination of simulated annealing, genetic algorithm, particle swarm optimisation, fuzzy theory, neural network, ant colonies, and tabu searching to address the out of service problem. The objective was to reduce the out of service areas and the number of manually and automated controlled switch operations and losses. However, the extensive computational time when applied to practical distribution systems constitute a noticeable drawback of this method.

A concise review of the different restoration methodologies highlighting the state of the arts along with the merits and demerits of the various approaches have been examined in the literature. Considerable research has been conducted in the areas of DERs applications in distribution networks as well as for microgrid restoration. However, at present there seems to be a dearth of sufficient information about the application of DERs in power system BSR unlike other sectors like the transport sector. As observed by [32], specific research in the area of black start from distribution systems and microgrids is quite limited. This study therefore fills that knowledge gap by implementing a BSR using DERs to stabilise the grid. Furthermore, with the consistent power grid outages frequently reported in Nigeria, besides promoting power grid decarbonisation, the study demonstrates knowledge advancement in grid resilience by the application of coordinated optimal route restoration strategy deployed in this study. Unlike existing methods, our formulated restoration approach priorities the restoration route and choses the route with least voltage and frequency drop. Different load restoration scenarios have been explored using data from a real power network in Nigeria. The mathematical formulation of the BSR problem as a dynamic optimisation problem is presented next, followed by the introduction of the MILP model which is

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used to linearise the original nonlinear dynamic optimisation problem. Lastly, the solution methodology of the proposed modelled BSR method is also presented.

III. Mathematical formulation

A BSR mathematical formulation based on dynamic optimisation has been adopted in this study. As expressed in [33], the issue of decision making over multiple time-step is addressed using this method. The discrete-time dynamic optimisation problem is expressed in equation (1) as;

$$max\ or\ min \sum_{Z_0}^{Z_N = \frac{T}{\Delta t}} \tilde{F}[Z_t, x(Z_t), u(Z_t)]\Delta t \quad (1)$$

The total number of time steps is assumed to be N, then $N = T/\Delta t$, where Δt is the length of each time step. $x(z_t)$ is defined as the state variable at time step z_t, and $u(z_t)$ as the control variable at time step z_t, $\tilde{f}[t,\ x(z_t),\ u(z_t)]$ as the transition equations determined by time step z_t, state variable $x(z_t)$ and control variable $u(z_t)$, and $\tilde{F}[z_t\ , x(z_t), u(z_t)]$ as the objective function.
According to [32], the time, state, and control variables must be defined in order to formulate the BSR problem. Hence, mathematically, the BSR problem is formulated as:

$$\min F[\mathbf{x}(t), \mathbf{u}(ţ)] \quad (2)$$
$$\text{s. t. } G_i[\mathbf{x}(t), \mathbf{u}(ţ)] \leq 0,\ i = 1, \dots, m \quad (3)$$
$$H_j[\mathbf{x}(t), \mathbf{u}(ţ)] = 0,\ j = 1, \dots, n \quad (4)$$
$$t \in \{Z_0, Z_1, Z_2,, \dots, Z_N,\}, \quad (5)$$

where (t) represent the vector of state variables at each time instant; (ţ) represent the decision variables at each time instant; $[(t), (ţ)]$ represents the objective function of the BSR problem, as a function of (t) and (ţ). Equations (3) and (4) represents m inequality constraints, which may be magnitude limits for node voltage, capacity limits for DGs, and n equality constraints such as power flow constraints respectively.

3.1.Objective function

The objective function of the BSR problem can be formulated as:

$$\text{Max} \sum_{ţ \in ſ} f\left(\boldsymbol{P}_{l,t}^{L}, \boldsymbol{P}_{g,t}^{\emptyset}, \boldsymbol{Q}_{g,t}^{\emptyset}, x_{ij,t}^{BR}, x_{l,t}^{L}\Delta t\right), \quad (6)$$

where ţ ∈ ſ ≔ {1,2, … , T} represents the set of steps, and T is the length of the horizon; $\boldsymbol{P}_{l,t}^{L}$ is a vector which represents the three-phase load demand on node l at step t; $\boldsymbol{P}_{g,t}^{\emptyset}$ is a vector representing the three-phase active power generated by a DG on node g at time step t; $\boldsymbol{Q}_{g,t}^{\emptyset}$ is a vector that represents the three-phase reactive power generated by a DG on node g at time step t; $(x_{ij,t}^{BR})$ is the switching state of a switchable line between node i and j respectively, at time step t; $x_{l,t}^{L}$ is the energisation level of a load at node l at time step t; Δt is the time span between two successive steps; (■) is the function that represents the restored load (e.g., in kW) as a consequent of the control variables. Hence, equation (6) represents the total amount of load restored at all the considered steps; i.e., the total restored energy (e.g., in kWh) over the considered timeframe or horizon.

3.2.Operational constraints and MILP formulation

Operational constraints presents power flow and other physical limits. Due to the unpredictability of any restoration process in practical systems, various assumptions are often made. Since the objective function is usually defined to maximise the total energy restored for each step considering the weight factor of each load l (denoted as β_l^L), we have:

$$\text{Max} \sum_{l \epsilon \mathcal{L}} \sum_{t \epsilon J} \sum_{\phi \epsilon \{a,b,c\}} \beta_l^L . p_{l,t}^{\emptyset} \Delta t \quad (7)$$

and subject to the following constraints; Initial condition constraints; DG current unbalance constraints; Linear three-phase power flow constraints; Topological constraints; Other constraints: voltage limit constraints, line and transformer capacity constraint, ramp rate constraints, DG output constraints, spinning reserve constraints, maximum load step constraints.
To capture all the variables used for the proposed simulation, the optimisation function is expressed in equation (8) as;

$$\beta_l^L . p_{l,t}^{\emptyset} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} + 0x_{ij,t}^{BR} + 0x_{l,t}^{L} \quad (8)$$

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Assuming a balanced three-phase distribution system, and the power flow model taking into consideration the energisation status of each line at each time step, the power balance constraints for each bus at each step is formulated as;

$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} - Me_{ij}^{\emptyset} x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq -\{(T) + Me_{ij}^{\emptyset} + \tilde{Z}_{ij} S_{ij}^{*} + \tilde{Z}_{ij}^{*} S_{ij}\} \quad (9)$$

$$\boldsymbol{U}_{i,t} - \boldsymbol{U}_{j,t} \leq \check{Z}_{i,j} \boldsymbol{S}_{ij,t}^{*} + \check{Z}_{ij}^{*} \boldsymbol{S}_{ij,t} + \mathrm{M}(1 - x_{ij,t}^{BR}) \boldsymbol{e}_{\boldsymbol{ij}}^{\emptyset}, \quad (10)$$

$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} + Me_{ij}^{\emptyset} x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq (T) - \left(\tilde{Z}_{ij} S_{ij}^{*} + \tilde{Z}_{ij}^{*} S_{ij} - Me_{ij}^{\emptyset}\right) \quad (11)$$

$$\boldsymbol{U}_{i,t} - \boldsymbol{U}_{j,t} \geq \check{Z}_{i,j} \boldsymbol{S}_{ij,t}^{*} + \check{Z}_{ij}^{*} \boldsymbol{S}_{ij,t} + \mathrm{M}(1 - x_{ij,t}^{BR}) \boldsymbol{e}_{\boldsymbol{ij}}^{\emptyset}, \text{ (i, j)} \in B/V, \mathrm{t} \in \mathcal{T}, \quad (12)$$

$$\sum_{h:(h,i)\in B} \boldsymbol{P}_{hi,t}^{BR} + \sum_{g:g=i,g\in G} \boldsymbol{P}_{g,t}^{\emptyset} = \sum_{j:(i,j)\in B} \boldsymbol{P}_{ij,t}^{BR} + \sum_{l:l=i,l\in\mathcal{L}} \boldsymbol{P}_{l,t}^{L}, \quad (13)$$

$$\sum_{h:(h,i)\in B} \boldsymbol{Q}_{hi,t}^{BR} + \sum_{g:g=i,g\in G} \boldsymbol{Q}_{g,t}^{\emptyset} = \sum_{j:(i,j)\in B} \boldsymbol{Q}_{ij,t}^{BR} + \sum_{l:l=i,l\in\mathcal{L}} \boldsymbol{Q}_{l,t}^{L}, \quad (14)$$

$$(i,j) \in B, \emptyset \in \emptyset_{i,j} \mathrm{t} \in \mathrm{f} \quad (15)$$

where $\mathrm{T} = \left(U_{i,t} - U_{j,t}\right), \boldsymbol{e}_{\boldsymbol{ij}}^{\emptyset} \in \mathbb{Z}_2^3$ is the vector with binary entries to represent the phases; $\boldsymbol{e}_{\boldsymbol{ij}}^{\emptyset} = [0,1,0]^T$ assuming a single-phase line for branch (i, j) (e.g., B-phase), then; $\boldsymbol{S}_{ij,t} = \boldsymbol{P}_{ij,t}^{BR} + \boldsymbol{Q}_{ij,t}^{BR}$ is the vector of three-phase apparent power flowing from bus i to j through line (i, j) at step t; $(\boldsymbol{P}_{l,t}^{L} + \boldsymbol{Q}_{l,t}^{L})$ is the three-phase load demand on node i at step t. Equations (10) and (12) ensures that the constraints are applied only to energised lines, except for voltage regulators and transformers. M, which is a big number, should be carefully selected to ensure that the constraints are only valid when the line is energised.

To model the behaviour of load demands at the various time steps, equations for the three-phase cold load pick-up (CLPU) is formulated as;

$$\boldsymbol{P}_{l,t}^{L} = \boldsymbol{P}_{l}^{L} \odot \left(\mathbf{S}^{\mathrm{u}} x_{l,t}^{L} - \Sigma_{k=1}^{t} \Delta \boldsymbol{P}_{l}(k) x_{l,t-k+1}^{L}\right), l \in \mathcal{L}, \mathrm{t} \in \mathrm{f}, \quad (16)$$

$$\boldsymbol{Q}_{l,t}^{L} = \boldsymbol{Q}_{l}^{L} \odot \left(\mathbf{S}^{\mathrm{u}} x_{l,t}^{L} - \Sigma_{k=1}^{t} \Delta \boldsymbol{Q}_{l}(k) x_{l,t-k+1}^{L}\right), l \in \mathcal{L}, \mathrm{t} \in \mathrm{f}, \quad (17)$$

where $\boldsymbol{P}_{l}^{L}, \boldsymbol{Q}_{l}^{L}$ are pre-outage three-phase active and reactive power of load l; $\mathbf{S}^{\mathrm{u}}$ is the vector representing undiversified loading factor for load l.

The connectivity constraints are mathematically modelled as a set of equality and inequality constraints to describe the physical connections of each component with the system. The mathematical representation of these rules is formulated by the following constraints:

$$x_{g,t}^{G} \leq s_{g,t}^{N}, \mathrm{g} \in \mathrm{G}/(\mathrm{G^s} \cup \mathrm{G^F}), \mathrm{t} \in \mathrm{f} \quad (18)$$

$$x_{g,t}^{G} - x_{g,t-1}^{G} \geq 0, \mathrm{g} \in \mathrm{G}, \mathrm{t} \in \mathrm{f}, \mathrm{t} > 1 \quad (19)$$

$$x_{g,t}^{G} = 1, \mathrm{g} \in \mathrm{G^s}/\mathrm{G^F}, \mathrm{t} \in \mathrm{f} \quad (20)$$

The starting of a dispatchable DG without black start capability only when it is connected to an energised node is facilitated by equations (18) and (19) respectively and once a DG is started, it cannot be tripped in the following steps. Equation (20) guarantees the starting of the substation node and the black start DGs at first step.
The black start DGs capability to regulate voltage is formulated as:

$$U_{g,t} = 1.0, \mathrm{g} \in \mathrm{G^s}/\mathrm{G^F}, \mathrm{t} \in \mathrm{f} \quad (21)$$

A simple matrix capturing the power flow model incorporating both the inequality and equality constraints is presented.

$$\begin{bmatrix} \sum_{l:l=i,l\in\mathcal{L}} k_{PL} & -\sum_{g:g=i,g\in\mathcal{G}} k_{PG} & 0 & 0 & 0 \\ 0 & 0 & \sum_{g:g=i,g\in\mathcal{G}} k_{QG} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{l,t}^{L} \\ p_{g,t}^{\emptyset} \\ Q_{g,t}^{\emptyset} \\ x_{ij,t}^{BR} \\ x_{l,t}^{L} \end{bmatrix} = \begin{bmatrix} \sum_{h:(h,i)\in B} p_{hi,t}^{BR} - \sum_{j:(i,j)\in B} p_{ij,t}^{BR} \\ \sum_{l:l=i,l\in\mathcal{L}} Q_{l,t}^{L} - \sum_{h:(h,i)\in B} Q_{hi,t}^{BR} + \sum_{j:(i,j)\in B} Q_{ij,t}^{BR} \\ s_{l,t}^{N} \\ s_{i,t}^{N} \\ s_{j,t}^{N} \end{bmatrix}$$

Bounds:

$$0 \geq p_{l,t}^{L} \leq P_{load_max}$$
$$x_{g,t}^{G}.P_{gen_min} \geq p_{g,t}^{\emptyset} \leq P_{gen_max}$$
$$x_{g,t}^{G}.Q_{gen_min} \leq Q_{g,t}^{\emptyset} \leq Q_{gen_max}$$

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$0 \geq x_{ij,t}^{BR} \leq 1$
$0 \geq x_{l,t}^{L} \leq 1$
$0 \geq k_{PL} \leq 1$
$0 \geq k_{Pg} \leq 1$
$0 \geq k_{Qg} \leq 1$

where k_{PL} is the constant term for the active load constraint; k_{Pg} is the constant term for the black start DGs active power constraint; k_{Qg} is the constant term for the black start DGs reactive power constraint.

3.3.Logic Constraints

The logic constraints are used to express the relationship between sequence of lines and buses. It is expressed mathematically as;

$$x_{ij,t}^{BR} \leq s_{i,t}^{N}, (i,j) \in \mathfrak{B}^{S}/\mathfrak{B}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (22)$$
$$x_{ij,t}^{BR} \leq s_{j,t}^{N}, (i,j) \in \mathfrak{B}^{S}/\mathfrak{B}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (23)$$
$$x_{ij,t}^{BR} = s_{i,t}^{N}, (i,j) \in \mathfrak{B}/(\mathfrak{B}^{S} \cup \mathfrak{B}^{F}), \mathfrak{t} \in \mathfrak{f} \quad (24)$$
$$x_{ij,t}^{BR} = s_{j,t}^{N}, (i,j) \in \mathfrak{B}/(\mathfrak{B}^{S} \cup \mathfrak{B}^{F}), \mathfrak{t} \in \mathfrak{f} \quad (25)$$
$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} - x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq \mathfrak{h} \quad (26)$$
$$x_{ij,t}^{BR} - \mathfrak{h} \geq 0, (i,j) \in \mathfrak{B}^{S}/\mathfrak{B}^{F}, \mathfrak{t} \in \mathfrak{f}, \mathfrak{t} > 1. \quad (27)$$

where $\mathfrak{h} = -x_{ij,t-1}^{BR}$

If a switchable line is energised, then both end nodes of a switchable line are energised and equations (22) and (23) ensures this is executed. Equations (24) and (25) guarantees the immediate energisation of non-switchable lines once one of the end nodes is energised. Equation (27) makes provisions such that once a line is energised, it cannot be tripped.

The energisation status of the damaged or non-damaged participating components is described by the initial condition constraints. The conditions describing the initial condition constraints is expressed as:

$$x_{ij,t}^{BR} = 0, (i,j) \in \mathfrak{B}^{S}/\mathfrak{B}^{F}, \mathfrak{t} = 1. \quad (28)$$
$$x_{g,t}^{G} = 0, g \in \mathfrak{G}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (29)$$
$$x_{e,t}^{ESS_CH} + x_{e,t}^{ESS_DISCH} = 0, \ e \in \mathcal{E}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (30)$$
$$x_{l,t}^{L} = 0, l \in \mathcal{L}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (31)$$
$$x_{ij,t}^{BR} = 0, (i,j) \in \mathfrak{B}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (32)$$
$$s_{i,t}^{N} = 0, i \in \mathfrak{N}^{F}, \mathfrak{t} \in \mathfrak{f} \quad (33)$$

During the first time step, all lines are assumed to be kept opened and then the non-participating DGs as well as energy storage systems (ESS) are kept de-energised. A non-energised load remains de-energised and a damaged or disconnected line continues to be de-energised while nodes that are damaged stays de-energised.

For each DG, two consecutive steps is usually required and the DG ramp rate constraints would normally be expected to be within the acceptable range. The active and reactive power output of the energised DG shall be maintained within the rated capacity. These constraints is described as:

$$\xi \leq \sum_{\phi \in \{a,b,c\}} P_{g,t}^{\phi} - \sum_{\phi \in (a,b,c)} P_{g,t-1}^{\phi} \leq x_{g,t}^{G} . P_{g}^{RAMP} . \Delta t, \mathfrak{g} \in \mathfrak{G}, \mathfrak{t} \in \mathfrak{f}, \mathfrak{t} > 1, \quad (34)$$
$$x_{g,t}^{G} . P_{g}^{min} \leq \sum_{\phi} P_{g,t}^{\phi} \leq x_{g,t}^{G} . P_{g}^{max}, \quad (35)$$
$$x_{g,t}^{G} . Q_{g}^{min} \leq \sum_{\phi} Q_{g,t}^{\phi} \leq \mathfrak{Z}, \mathfrak{g} \in \mathfrak{G}, \mathfrak{t} \in \mathfrak{f} \quad (36)$$

where $\xi = -x_{g,t}^{G} . P_{g}^{RAMP} . \Delta t$, $\mathfrak{Z} = x_{g,t}^{G} . Q_{g}^{max}$, P_{g}^{RAMP} is the maximum ramp rate for DG g; P_{g}^{min} is minimum value of active output of DG g; P_{g}^{max} is maximum value of active output of DG g; Q_{g}^{min} is minimum value of reactive output of DG g; Q_{g}^{max} is maximum value of reactive output of DG g.

The output of the de-energised DG is usually forced to zero by these constraints. For dispatchable DG operating at fixed power factor, an optional constraint can be added thus;

$$\tan(\arccos\varphi_{g}) . \sum_{\phi} P_{g,t}^{\phi} - \sum_{\phi} Q_{g,t}^{\phi} = 0, g \in \mathfrak{G}/\mathfrak{G}^{s}, \mathfrak{t} \in, \mathfrak{f} \quad (37)$$

where φ_{g} is the fixed power factor for DG g. This constraint does not apply to the substation node or the black start DG.

3.4.Topological Constraints

The series of rules, which describes the feasibility of a BSR sequence, is defined by the topological constraints. Adopting the tree topology, which is synonymous with radial topology, operated by most distribution system, the mathematically translation of these rules is represented by the following constraints:

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$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} - \sum_{ij} x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq \sum_{ki} x_{ki,t}^{BR} \quad (38)$$

$$s_{i,t}^{N} \leq \sum_{i:\,(i,j)\in\mathcal{C}} x_{ij,t}^{BR} + \sum_{i:\,(k,i)\in\mathcal{C}} x_{ki,t}^{BR}, \mathrm{i} \in \mathcal{K}, \mathrm{t} \in \mathrm{f} \quad (39)$$

$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} - \sum_{ij} x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq -\sum_{ij} x_{ij,t-1}^{BR} - \sum_{ij}(\mathrm{C}) + 1 + Ms_{i,t-1}^{N} \quad (40)$$

$$\sum_{i:\,(i,j)\in\mathcal{C}} (x_{ij,t}^{BR} - x_{ij,t-1}^{BR}) + \sum_{i:\,(k,i)\in\mathcal{C}} \mathrm{O} \leq 1 + Ms_{i,t-1}^{N}, \mathrm{i} \in \mathcal{K}, \mathrm{t} \in \mathrm{f}, \mathrm{t} > 1 \quad (41)$$

$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} + x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq s_{i,t-1}^{N} + s_{j,t-1}^{N} \quad (42)$$

$$x_{(i,j),t}^{BR} \leq s_{i,t-1}^{N} + s_{j,t-1,}^{N} (i,j) \in \mathcal{B}^{s}, \mathrm{t} \in \mathrm{f}, \mathrm{t} > 1 \quad (43)$$

$$0p_{l,t}^{L} + 0p_{g,t}^{\emptyset} + 0Q_{g,t}^{\emptyset} + x_{ij,t}^{BR} + 0x_{l,t}^{L} \leq \left(s_{i,t}^{N} - s_{i,t-1}^{N}\right) + \left(s_{j,t}^{N} - s_{j,t-1}^{N}\right) - x_{ij,t-1}^{BR} \quad (44)$$

$$(s_{i,t}^{N} - s_{i,t-1,}^{N}) + (s_{j,t}^{N} - s_{j,t-1,}^{N}) \geq x_{(i,j),t}^{BR} - x_{(i,j),t-1}^{BR}, (i,j) \in \mathcal{B}^{\mathrm{s}}, \mathrm{t} \in \mathrm{f}, \mathrm{t} > 1 \quad (45)$$

where $\mathrm{C} = x_{ki,t-1}^{BR} - x_{ki,t-1}^{BR}$ and $\mathrm{O} = x_{ki,t}^{BR} - x_{ki,t-1}^{BR}$

IV. Results and discussions

4.1.Description of model

The proposed BSR solution method adopted, modelled, and presented in this study is used to simulate the performance of the power system in a black out event. The Nigerian 48-bus system was chosen as a case study to assess the system performance. The frequent unscheduled blackout events and load shedding the country has continued to experience as reported in [2, 34 - 39] necessitated its choice. The modelling of the test system and the various simulated results of the load restoration performance analysis is succinctly discussed.

The line diagrams, various data set, and related system parameters used in the modelling of the Nigerian 48-bus system were obtained from the Transmission Company of Nigeria (TCN). The transmission system currently comprises of 5,523.8 km of 330 kV, 6801.49 km of 132 kV, 32 substations of 330/132 kV with total installed transformer capacity of 7688 MVA [39]. The average available capacity on the 330/132 kV is 7,364 MVA. Using the network parameters obtained, and by making necessary inference, the sectional model of the 48-bus system was built in NEPLAN (Fig. 1). The study assumes that PV installations closest to the buses are considered to participate directly in the BSR, while the PV installations that are farther from the buses are considered as energy reserves, hence deployed as an additional generating capacity of the region. The projected load switching possibility in terms of switching time of the various participating generators in the network is presented in Table 1.

Table 2 captures the possible switching routes of the various loads on the different transmission substation (TS) buses by participating generators in the network.

Table 1. Possible switching time of load by generators.

S/N	Bus	Name	Time step from generator to load considering the entire network (min)													
			G 1	G 2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12	G13	G14
1	3	Jebba TS	2	2	4	2	3	6	4	6	8	8	4	6	7	7
2	6	Oshogbo TS	3	3	3	3	2	5	3	5	7	8	3	5	6	6
3	7	Ayede TS	4	4	2	4	3	7	4	6	8	9	4	6	7	7
4	8	Ikeja West TS	4	4	2	4	3	5	2	4	6	7	2	4	5	5
5	10	Ganmo TS	3	3	4	3	3	7	4	6	8	9	4	6	7	8
6	11	Kaduna TS	4	4	6	2	4	5	6	6	8	8	6	6	7	7
7	12	Katampe TS	4	4	6	2	4	5	6	6	8	8	6	8	7	7
8	13	Gwagwalada TS	4	4	6	2	5	4	6	5	6	8	6	5	6	6
9	18	Okearo TS	5	5	5	3	5	4	5	3	4	5	7	2	5	5

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10	19	Benin TS	6	6	4	6	5	3	4	2	4	6	2	2	3	3
11	21	Jos TS	5	5	7	3	6	7	7	7	5	6	7	7	5	6
12	22	Lokoja TS	5	5	7	2	6	3	6	4	6	8	4	4	5	5
13	24	Onitsha TS	7	7	5	7	6	4	5	3	3	5	3	3	2	2
14	25	Ajaokuta TS	6	6	5	4	6	2	5	3	5	7	3	3	4	4
15	28	Makurdi TS	6	6	8	4	7	7	8	6	4	5	6	6	4	5
16	30	New Heaven TS	8	8	6	6	7	5	6	4	4	5	4	4	3	3
17	34	Aladja TS	8	8	6	8	7	5	6	2	6	8	4	2	5	8
18	35	Ugwuaji TS	7	7	7	5	8	6	6	5	3	4	5	5	3	4
19	39	Ikot Ekpene TS	8	8	6	5	8	6	7	5	2	3	5	5	2	4
20	40	Adiabor TS	9	9	9	9	10	8	9	7	3	2	7	6	3	5

Table 2. Possible switching of load L6 on Oshogbo TS bus by participating generators.

S/N	Name	Lines from Generators to Load 6 (Oshogbo TS) considering the entire network
1	G1	L34, L36
2	G2	L23, L36,
3	G3	L714, L67
4	G4	L35, L36
5	G5	L69
6	G6	L2533, L2235, L1322, L513, L35, L36 L2533, L1925, L1619, L816, L68
7	G7	L817, L68
8	G8	L2627, L1927, L1619, L816, L68 L1926, L1619, L816, L68
9	G9	`L3238, L2432, L1924, L1619, L816, L68
10	G10	L3839, L3539, L2835,L2128, L1121, L511,L35, L36 L3238,L2432,L1924,L1619,L816,L68, L36
11	G11	L816, L68 L1618,L818,L68
12	G12	L1927,L1619, L1618, L816,L68 L2734, L2627, L1925, L1619, L816,L68
13	G13	L2432, L1924, L1619, L816,L68
14	G14	L2431,L1924,L1619,L816,L68

Table 3. Oshogbo region performance characteristics on loading.

S/N	Bus no.	Load bus	Voltage (p.u.)		Frequency (dip from 50 Hz)	
			Maximum drop	Settling value	Maximum dip	Settling value
1	6a	Oshogbo TS	0.3844	0.8545	-1.1785	-0.3133
2	6d	Oshogbo (alternate)	0.7127	0.8597	-1.6969	-1.5208
3	7	Ayede TS	0.6299	0.8836	-1.9058	-1.3722
4	10	Ganmo TS	0.5772	0.8542	-0.5134	-0.0750
5	8	Ikeja West TS	0.6570	0.8985	-1.8568	-1.2620
6	18	Okearo TS	0.9475	1.0136	-0.5653	-0.3217

The switching sequence of the various loads during restoration takes cognisant of the anticipated drops in voltage and frequency for all the restoration route and choses the route with least voltage and frequency drop. Also, participating transmission lines routes taken to switch the various desired

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loads takes into consideration the optimal restoration time for the load which is very essential as well as the quality of the power supply during power system restoration.

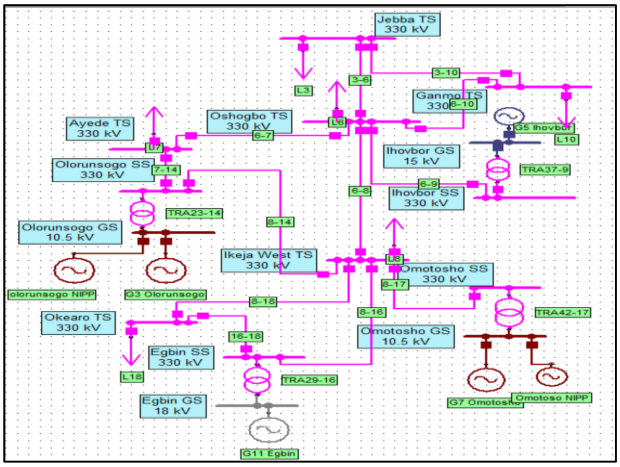

Fig. 1. Sectional model of Nigerian 48-bus system showing Oshogbo region.

The spot active and reactive loads for Oshogbo region based on the optimal switching status taking into consideration the nearest route to restoration is presented in Fig. 2 and Fig. 3. The primary objective of the BSR is to ensure the immediate return of normalcy and stability to the power system, since the power system must meet its demand for reactive power. As seen in the active and reactive power plots presented, the restored load begins to normalise, as it approaches the 10th time step.

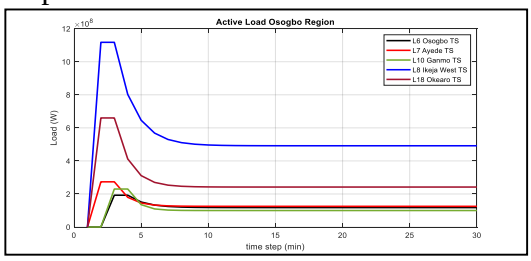

Fig. 2. Active loads in Oshogbo region.

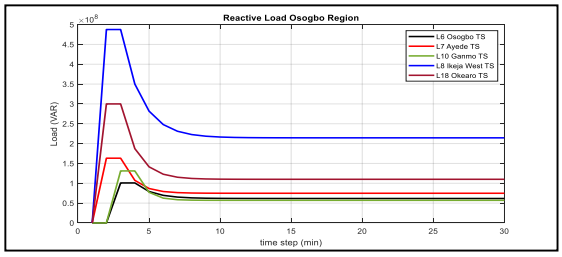

Fig. 3. Reactive loads in Oshogbo region.

The possible switching pattern for the load at Oshogbo presented in Table 3 is plotted in Fig. 4 and Fig. 5 respectively for possible black start restoration from four (4) DGs. The restoration from Ayede TS provide the most accommodating transient condition with a drop of 1.176 Hz and settling with a drop of 0.290 Hz. The model considers the reduction in restoration time while avoiding the compromise of system parameters. If a possible compromise in system parameters of voltage and frequency is anticipated, a supporting black start DGs will be deployed, but a recorded increase in the time step before restoration will be activated as well. The MILP adopted in this study presents the most optimal restoration routes following a blackout occurrence for every load, as demonstrate in Fig. 4 and Fig. 5 respectively where Ayede TS is the most optimally suiting restoration route for Oshogbo region.

The nominal system frequency in Nigeria is 50 Hz, however, a narrow operating band of +/- 0.5% (49.75 – 50.25 Hz) and +/- 2.5% (48.75 – 51.25 Hz) is permissible during normal and under system stress conditions respectively. Throughout the loading phase, the performance characteristics of the network parameters was closely monitored to ensure that the stipulated frequencies limits were not exceeded, hence maintaining the security and integrity of the transmission system.

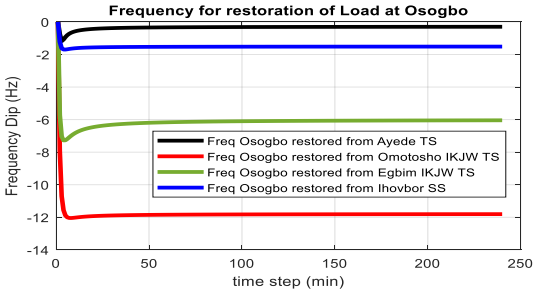

Fig. 4. Frequency dip during Osogbo Load black start restoration from different TS.

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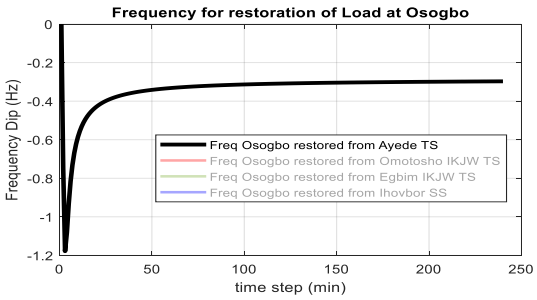

Fig. 5. Frequency dip during Osogbo Load black start restoration from Ayede TS.

V. Conclusion

In this study, a BSR framework based on DERs is developed and used to stabilise the frequency and voltage response of the power system during black start. It was demonstrated that the developed method is capable of implementing BSR effectively, since load restoration could be sequentially initiated at various times in different sections of the network. Without the applied method, switching can only be done considering optimisation of switching time, but with the applied method, the path with the least anticipated voltage and frequency drop is taken into consideration in the choice of the restoration route. The use of DERs as black starts services, the application of DERs to improve power system resilience and in the development of an effective restoration strategy is demonstrated.

The simulated results show that it is possible to use DERs to stabilize the grid frequency. The various results indicating the contributions of DERs to grid stabilisation were obtained, and tabulated on the various loading performance Tables. When considering the whole power system, the findings demonstrate that although the frequency dips, it however quickly returns to the stipulated operating limit and stabilises itself back to the normal operating limits of 50 Hz resulting from DERs participation in the grid frequency stabilisation. The results of this study addresses an important gap in the state of the art especially in the part of the world where blackouts are a reoccurring concern.

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